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A Method for Estimating Mortality Rate and Divers' Sighting Rate for Tagged Abalones

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This paper proposes a method for estimating the mortality rate and divers' sighting rate by taking the effect of movement of abalones into consideration, from the data of the number of alive and dead abalones counted by divers at survey day and the initial number of tagged abalones. The abalones are classified into four states by the combination from the elements as alive or dead and observable or unobservale. The probabilities for the states at survey day t are presented by a stationary Markov chain. The probabilities which an abalone is sighted alive or dead at t are given by the products of the probabilities of the divers' sighting and the probabilities for the alive or dead states in the observable. The log-likelihood function L is obtained from the multinominal distribution. The unknown parameters are estimated by maximizing L. The variance-covariance matrix of the estimates is approximately given by the inverse of the information matrix. The values of the parameters for maximizing L are found by Davidon's variable metric method. Furthermore, the method is applied to the data of Japanese black abalone *Haliotis discus discus* in the coast off Chiba Prefecture. The estimates of the mortality rate per day are about 3.2% for group 1 (10–24 mm) and 0.64% for group 2 (22–30 mm). The true values of the sighting rate for alive abalones are 41% at one day after releare, 23% at six, 20% at fourty five for group 1, and 37%, 15% and 13% for group 2, respectively. The limitations of the application and the modification of the method are discussed.

In order to estimate the mortality rate of abalones, many tagging or marking experiments have been carried out. There are two kinds in the experiments. One is designed to estimate the rate from data of recapture[1,2] and the other is to do from data of number of the abalones sighted by divers.[3-6]

The latter kind of experiment would be available for young abalone which are small and unexploitable in commercial fisheries. Some methods of estimation for the abalones from the data of the latter experiment were proposed.[4-6] But the methods need tags with a code number for the sake of individual identification and the application to abalones of small size may be rather difficult. Then, in this paper we consider the tagging experiment without code number.

In the experiment, after release of the abalones, divers search them in the area around the released point and count alive and dead abalones, and the sighting surveys are repeated.

The divers would not find all the abalone in the serching area because abalone inhabit complicated rocky reefs and because the abalone in crevices, caves and under big stones are difficult to be sighted. There are observable and unobservable places in the area. If the divers' sighting rate ((number of sighted abalone)÷(total)) is constant irrespectively of time, the estimation of the mortality rate would not be affected by the rate.

However the sighting rate may be changed depending upon the time because of movement of the abalones. Even if the released point is observable for the divers, some of the abalone would move to the unobservable. The ratio of the abalone on the observable place to the unobservable would be affected by the movement. A changes in the ratio varies the sighting rate and leads to some bias in the estimation of the mortality rate.

The purpose of this paper is to present a method for estimating the mortality rate and divers' sighting rate by taking the effect of movement of abalone into consideration, from sighting data of tagging experiment. Furthermore, the

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method is applied to the practical data. The limitations of the application and the modification of the method are discussed.

Method

The alive abalone would repeat to emigrate and immigrate between the two places (observable and unobservable). Also the dead abalone may be may be transported from one to the other by water flow, etc. The shells of the dead abalone and the tags may be crushed by predators. In this case, a decrease in the number of the tagged individuals can be treated as transportation into the unobservable. The immigration or emigration rates of alive abalone are considered to be different from the transportation rate of dead abalone.

The divers would not find all the alive and dead abalones in the observable place and there may be some differences of the divers' sighting rate between the alive and dead abalone. The divers would easily sight the upside-down shells of the dead abalone in comparison with the alive abalones, whereas the crushed shells are difficult to be sighted.

Then, the abalone are classified into four states from the combination of alive or dead and observable or unobservable, and the state variables for the abalone are denoted by $X_i (i=1, 2, 3, 4)$, as shown in Table 1.

Table 1. Classification of state variables for tagged abalones

State variable	Alive/Dead	Place
X_1	Alive	Observable
X_2	〃	Unobservable
X_3	Dead	Observable
X_4	〃	Unobservable

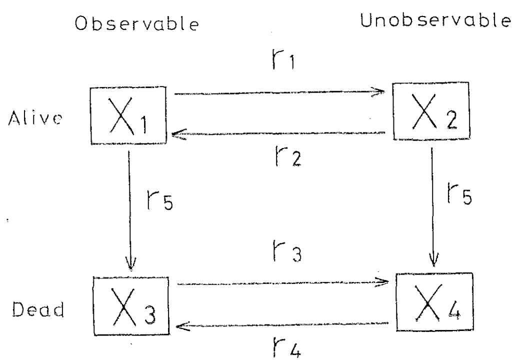

Fig. 1. Relationship among the state variables for abalone X_i $(i=1, 2, 3, 4)$. The notations r_i $(i=1, \cdots, 5)$ in figure show the transition probabilities from one state X_i to the other state X_j.

Let $p_i(t)$ be the probability of X_i for an abalone at day $t (t=1, 2, ..)$. The probability vector $P(t)$ is defined by

$$P(t)=(p_1(t), p_2(t), p_3(t), p_4(t))' \qquad (1)$$

where notation " ' " shows the transpose and $p_i(t)$ satisfy $\sum_{i=1}^{4} p_i(t)=1$.

Let r_1, r_2, r_3 and r_4 $(0 \leqq r_i \leqq 1)$ be the transition probabilities for a day from X_1 to X_2, from X_2 to X_1, from X_3 to X_4 and from X_4 to X_3, respectively. The mortality rate per day for the abalone is assumed to be constant irrespectively of the places, denoted by r_5 $(0 \leqq r_5 \leqq 1)$ (Fig. 1). Using these notations, the transition probability matrix R is presented by

$$R=\begin{pmatrix} 1\text{-}r_1\text{-}r_5 & r_2 & 0 & 0 \\ r_1 & 1\text{-}r_2\text{-}r_5 & 0 & 0 \\ r_5 & 0 & 1\text{-}r_3 & r_4 \\ 0 & r_5 & r_3 & 1\text{-}r_4 \end{pmatrix}. \qquad (2)$$

From Eqs. (1) and (2), $P(t)$ is represented by

$$P(t)=R\,P(t\text{-}1) = R^t\,P(0) \qquad (3)$$

where $P(0)$ shows the initial probability vector:

$$P(0)=(p_1(0), p_2(0), p_3(0), p_4(0))' \qquad (4)$$

and $p_i(0)$ shows the probability of X_i for an abalone at $t=0$ $(0 \leqq p_i(0) \leqq 1, \sum_{i=1}^{4} p_i(0)=1)$. The values of $p_i(0)$ can be determined from the way to release the tagged abalones. The values of $P(t)$ are calculated from the stationary Markov chain[7] characterized by Eqs. (1)~(4).

Let $\pi_1(t)$, $\pi_2(t)$, and $\pi_3(t)$ be the probability for an alive abalone to be sighted, for a dead individual to be sighted and the probablity for an individual not to be sighted. By assuming that the sighting effort is constant, these are given by

$$\pi_1(t)=r_6 p_1(t) \qquad (5)$$

$$\pi_2(t)=r_7 p_3(t) \qquad (6)$$

$$\pi_3(t)=1-\pi_1(t)-\pi_2(t) \qquad (7)$$

where r_i: probabilities for an alive $(i=6)$ or dead $(i=7)$ abalone in an observable place to be sighted $(0 \leqq r_i \leqq 1)$.

Next, we consider the log-likelihood function L to estimate $r_i (i=1, .., 7)$. Let N, $n(t_k)$ $(k=1, .., m)$ and $d(t_k)$ denote the total released number of tagged abalone, the number of the alive and dead abalones sighted by the divers at t_k, respec-

Table 2. Number of tagged abalones (*H. discus discus*) found by divers for two groups of size of shell from the survey report*1

Group	Alive/Dead/(k) t_k*2	(0) 0	(1) 1	(2) 2	(3) 4	(4) 6	(5) 8	(6) 18	(7) 45
1*3	Alive $n(t_k)$	471	198	121	130	102	46	67	18
	Dead $d(t_k)$	0	5	1	13	5	0	10	18
	Sum	471	203	122	143	107	46	77	36
2*4	Alive $n(t_k)$	475	178	120	106	68	38	42	81
	Dead $d(t_k)$	0	6	9	7	9	1	14	13
	Sum	475	184	129	113	77	39	56	94

*1 See footnote.
*2 Number of days from release.
*3 Small size (10~24 mm, mean 16.3).
*4 Large size (22~30 mm, mean 26.7).

tively. Under the condition of the random sampling at t_k, the probability which $n(=n(t_k))$ and $d(=d(t_k))$ individuals are sighted at t_k is presented by the following multinominal distribution:[8]

$$\frac{N!}{n!d!(N-n-d)!}\pi_1{}^n\pi_2{}^d\pi_3{}^{N-n-d}.$$

Because the samplings are considered to be independent each other, from the joint probability of the above equation, the following log-likelihood function is obtained:

$$L=\sum_{k=1}^{m}[n(t_k)\log\pi_1(t_k)+d(t_k)\log\pi_2(t_k)+\{N-n(t_k)-d(t_k)\}\log\pi_3(t_k)]. \quad (8)$$

The estimates of r_i, $\hat{r}_i$, are calculated by maximizing L. The variance-covariance matrix of $\hat{r}_i$, $\hat{V}_r$ is approximately given by the inverse of the information matrix:[8]

$$\hat{V}_r \simeq I^{-1} \quad (9)$$

where the element of I, say s_{ij}, is given by

$$s_{ij}=\sum_{k=1}^{n}\sum_{m=1}^{3}\left[\frac{N}{\pi_m(t_k)}\frac{\partial\pi_m(t_k)}{\partial r_i}\frac{\partial\pi_m(t_k)}{\partial r_j}\right]_{\hat{r}_i}. \quad (10)$$

An estimate $\hat{r}_i$ may be searched by the Davidon's variable metric method.[9] The domain of r_i is naturally (0, 1) but the calculated r_i may be out of the domain at any steps of the searching process. In such cases, after setting the boundary values (0 or 1) for r_i at the next step, the calculations are iterated.

Applications

Materials

The method is applied to the data* from the tagging experiment for Japanese black abalone *Haliotis discus discus* off Kominato, Chiba Prefecture, in 1989.

The abalone for tagging were stratified into two groups by size (group 1: 10–24 mm with mean 16.4, group 2: 22–30, 26.7). After the tags with simple code number were set to the shells of the abalone by a strong adhesive agent.

All the abalone were released on the center of the survey area of 30 m×30 m. Immediately after the release, the abalones not to be vigorous were collected. The practical number of the released abalone were 471 individuals for group 1 and 475 for group 2.

One of the authors, H. Yamakawa, and the divers made sighting surveys and counted the abalone in the area seven times. All the sighting surveys were started at noon for the sake of avoiding the effect of the daily migration of the abalones. The numbers of the alive and dead abalone sighted by the divers are shown in Table 2.

Results

Because the released point is observable for the divers, we set $P(0)=(1, 0, 0, 0)'$. For the sake of obtaining the appropriate initial setting values of r_i, the values of L in Eq. (8) were calculated for $3^7(=2187)$ sets of values of $r_i(=0.2$, 0.5 and 0.8). After the calculations, the searchings by the Davidon's variable metric method were carried out for each group. The calculations were stopped when the relative difference between the successive values of L in the iterative calculations is less than 10^{-8}.

Table 3 shows the values of the estimates, variances and coefficient of variation (CV). Table

* Japan Sea-Farming Association: Showa 63 Nendo Awabirui Houryu Shubyo No Shoki Genmo Gen-in Kaimei Chosa Hokokusho, Japan Sea-Farming Association, Tokyo, 1989, pp. 1–27.

Table 3. Estimates of parameter and variances for groups of abalone

Group	Statistic	L_{max}*1	r_1	r_2	r_3	r_4	r_5	r_6	r_7
1	Estimate	−1802.64	.18742	.12200	1.00000	.15519	.032133	.50250	.34347
	Variance	—	.00350	.00253	.26517	.00581	.000051	.00358	.02828
	CV*2	—	.31567	.41236	.51495	.49104	.22272	.11911	.48960
2	Estimate	−1828.44	.28886	.099497	.15356	.021350	.0064220	.52624	1.00000
	Variance	—	.00374	.000586	.02648	.000453	.0000267	.00468	.73714
	CV*2	—	.21163	.24325	1.0597	.99658	.80458	.12996	.85857

*1 Estimated values of maximum of L.
*2 Coefficient of variation.

Table 4. Estimated values of $p_i(t_k)$ ($i=1, 2, 3, 4$) for groups of abalone at survey day t_k from values in Table 3

Group	(i) $/t_k$	1	2	4	6	8	18	45
	(1)	0.780	0.632	0.460	0.373	0.325	0.219	0.091
	(2)	.187	.305	.418	.449	.445	.336	.139
1	(3)	.032	.025	.027	.032	.037	.064	.105
	(4)	.000	.038	.095	.146	.192	.381	.665
	Sum	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	(1)	0.705	0.525	0.349	0.283	0.257	0.228	0.192
	(2)	.289	.462	.625	.679	.693	.662	.557
2	(3)	.006	.010	.013	.014	.014	.018	.035
	(4)	.000	.003	.013	.024	.036	.091	.217
	Sum	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5. Estimated values of $\pi_j(t_k)$ ($j=1, 2, 3$) for groups of abalone

Group	(j) $/t_k$	1	2	4	6	8	18	45
	(1)	0.392	0.318	0.231	0.187	0.163	0.110	0.046
1	(2)	.011	.009	.009	.011	.013	.022	.036
	(3)	.597	.673	.760	.802	.824	.868	.918
	Sum	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	(1)	0.371	0.276	0.184	0.149	0.135	0.120	1.101
2	(2)	.006	.010	.013	.014	.014	.018	.035
	(3)	.623	.714	.803	.837	.851	.862	.864
	Sum	1.000	1.000	1.000	1.000	1.000	1.000	1.000

2 suggests the following points.

The monthly rate of the mortality calculated from the value of $\hat{r}_5$ in Table 3 for group 1 is about 62%, and is considerably large in comparison with that for group 2 (18%).

The difference of $\hat{r}_6$ between the groups is small, whereas that of $\hat{r}_7$ between the groups is large. The dead abalone of large size are easy to be sighted but the small-size dead are difficult.

The value of $\hat{r}_2$ for group 1 is close to that for group 2, but the value of $\hat{r}_1$ for group 2 is larger than that of group 1. The large abalones have rather crevicing behaviour than the smalls.

The values of $\hat{r}_3$ and $\hat{r}_4$ are 1.00 and 0.16 for group 1, and 0.15 and 0.02 for group 2. The value of $\hat{r}_3$ is greater than $\hat{r}_4$ and the difference for group 1 is larger than group 2. The number of dead abalone for group 1 is expected to be large but the deads are difficult to be sighted.

The data of group 1 give the better results (CV=0.12~0.51) than group 2 (0.13~1.06). But the values of CV of $\hat{r}_3$, $\hat{r}_4$ and $\hat{r}_7$ for both groups are larger in comparison with the others and excessive values of $\hat{r}_3$ of group 1 and $\hat{r}_7$ of group 2 may be given, which are almost 1. The estimates for the dead abalone may have some bias.

Tables 4 and 5 show the values of $p_i(t)$ and $\pi_i(t)$ calculated from Eqs. (1)~(4) and the values of $\hat{r_i}$ in Table 3. The sighting rates in this paper, r_6 and r_7, are apparent because the abalone in the unobservable place are not taken into consideration. The true values of the sighting rates should be defined by $\pi_1(t_k) \div \{p_1(t_k)+p_2(t_k)\}$ and $\pi_2(t_k) \div \{p_3(t_k)+p_4(t_k)\}$ instead of r_6 and r_7. From the values in Tables 4 and 5, the values of $\pi_1(t_k) \div \{p_1(t_k)+p_2(t_k)\}$ are 41% at $t=t_1$, 23% at t_4, 20% at t_7 for group 1, and 37%, 15% and 13% for group 2, respectively.

Discussion

The method has the merit of estimating the mortality rates at various sizes and also ages for abalone from the data of divers' sighting surveys. However the method has the following imperfections.

Firstly, the parameters in the model are assumed to be constant. The values of the parameters in R would be changed depending upon time. In this case, a method to modify the model is to introduce the time dependent R, for an example, R_1 for $t=0 \sim t_1$ and R_2 for $t=(t_1+1) \sim t_2$. The modification gives two problems. One is that the number of the parameters increases and the other is to determine the optimum number of R_j and values of t_j. For the first problem, if the number of the parameters are less than twice number of the survey days, they would be estimated (See Eq. (8)). However the practical number of R_j would be limited by the feasibility of the frequency of the surveys for resolving the first problem. The second problem of model selection would be mathematically solved by likelihood ratio,[10] AIC,[10] etc..

Secondly, there may be some differences of the values of r_5 between the observable and unobservable places because the unobservable place includes such sites as small caves, which are suitable for keeping off predators. The situation is mathematically presented by midifying of R into new matrix R' with two mortality rates, r_5 and r_5' and also such situations as the transition probability from X_1 to X_4 by predation may not be zero. Estimates of these rates can be obtained by the method as same as mentioned above.

Thirdly, by the mortality due to tagging and the shedding of tags, the value of r_5 is over-estimated. Because the tag set on the shell would give little demage to the body of the abalone, the mortality due to tagging would be negligible. Also because the tags are set to the abalones with a strong adhesive agent and the period of the surveys is short, the shedding of tags is considered to be negligible. However it may be necessary to examine the effect of the shedding of tags by the double tagging method because the magnitude of the effect is not estimated.

Fourthly, there is no gurantee of a unique solution. Then, the calculations for various initial setting values must be carried out.

Fifthly, the value of N must be large. The observation in the applications suggested the possibilities of bias in the estimates for the dead abalones. The fluctuation of $d(t)$ would cause the large variances because the values of $d(t)$ are small. If the value of N are increased, the values of $d(t)$ are expected to be large and the relative fluctuation of $d(t)$ would decrease. Many tagged abalones are needed.

Finally, the estimates of r_i would be changed depending upon the natural environment around the released point. For examples, the values of r_6 and r_7 are related to the shape of bottom, transparency, etc.. The values of r_5, r_6 and r_7 in the complicated bottom would be smaller than the flat bottom and the values of r_1 and r_3 in the complicated bottom may be larger than the flat bottom. Hence, a large-scale tagging experiment are necessary to obtain the unfied estimates for the total population.

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